

Surface subgroups on 1-vertex and 3-vertex polyhedra forming triangular hyperbolic buildings

Riikka Kangaslampi

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Abstract

In this article we study surface subgroups of groups acting simply transitively on vertex sets of certain hyperbolic triangular buildings. We concentrate on surface subgroups on 1-vertex and 3-vertex polyhedra that can be used to define these buildings. In [10] and [3] the authors constructed and classified all groups acting simply transitively on the vertices of hyperbolic triangular buildings of the smallest non-trivial thickness. These groups gave the first examples of cocompact lattices acting simply transitively on vertices of hyperbolic triangular Kac-Moody buildings that are not right-angled. They constructed the groups with the polygonal presentation method introduced in [15]. The hyperbolic buildings arise as universal covers of polyhedra glued together from with 15 triangular faces with words written on the boundary. In [11] the authors proved, that most of the obtained 23 torsion free groups do not admit periodic planes of genus 2. Here we take another approach to study surface subgroups of these groups. Namely, we consider first 2-cycles in the 1-vertex polyhedron defined by the triangles, then in the 3-vertex cover of this polyhedron. As a result we find surface subgroups only in three of the 23 torsion free groups. These were also found in [11].

1 Introduction

In [10] the authors classified all torsion-free groups acting simply transitively on the vertices of hyperbolic triangular buildings of the smallest non-trivial thickness. They constructed the groups with the polygonal presentation method introduced in [15]. As a result, they obtain 23 non-isomorphic groups, each defined by 15 generators x_1, x_2, \dots, x_{15} and 15 cyclic relations, each of them of the form $x_i x_j x_k = 1$, where not all the indices are the same. The underlying hyperbolic building is the universal cover of the polyhedron glued together from 15 geodesic triangles with angles $\pi/4$ and with the letters from the relations written on the boundary. In constructing the polyhedron the sides of the triangles with the same labels are glued together, respecting the orientation. For example, the presentation T_1 obtained in [10] are given in Table 1.

Thus these sets of 15 triangles, with angles $\pi/4$, words specified in [10] written at the boundary and glued together respecting orientation, all give a polyhedron that has one vertex and the smallest generalised quadrangle as the

(x_1, x_1, x_{10})
 (x_1, x_{15}, x_2)
 (x_2, x_{11}, x_9)
 (x_2, x_{14}, x_3)
 (x_3, x_7, x_4)
 (x_3, x_{15}, x_{13})
 (x_4, x_8, x_6)
 (x_4, x_{12}, x_{11})
 (x_5, x_5, x_8)
 (x_5, x_{10}, x_{12})
 (x_6, x_6, x_{14})
 (x_7, x_7, x_{12})
 (x_8, x_{13}, x_9)
 (x_9, x_{14}, x_{15})
 (x_{10}, x_{13}, x_{11})

Table 1: Triangles in presentation T_1 from [10].

link. The universal cover of this polyhedron is a hyperbolic triangular building, and the group with 15 generators x_1, x_2, \dots, x_{15} and the 15 words from the boundaries of the triangles as relations, acts on the building cocompactly and simply transitively. These groups are the first examples of cocompact lattices acting simply transitively on vertices of hyperbolic triangular Kac-Moody buildings that are not right-angled. For a general introduction to the theory of hyperbolic buildings and their lattices, see the survey [14] by A. Thomas.

Here we study the 23 groups further, motivated by Gromov's famous surface subgroup question: Does every one-ended hyperbolic group contain a subgroup which is isomorphic to the fundamental group of a closed surface of genus at least 2? Recall, that a group G is a *surface group* if $G = \pi_1(\mathcal{F})$, where \mathcal{F} is a closed surface. If in addition \mathcal{F} is finite, then $\pi_1(\mathcal{F})$ has one of the following forms (see [13])

- (i) $G = \langle a_1, b_1, \dots, a_n, b_n \mid a_1 b_1 a_1^{-1} b_1^{-1} \dots a_n b_n a_n^{-1} b_n^{-1} \rangle$, when \mathcal{F} is orientable and of genus n ,
- (ii) $G = \langle a_1, \dots, a_n \mid a_1^2 a_2^2 \dots a_n^2 \rangle$, when \mathcal{F} is non-orientable and of genus n .

Even though Gromov's question remains open, it is known that there are many classes of hyperbolic groups, for which the answer is positive. For example, see [5] for surface subgroups of right-angled Artin groups or [2], where Calegari and Walker show, that a random group contains many quasiconvex surface subgroups. Existence of surface subgroups in right-angled hyperbolic buildings was shown in [6], and in hyperbolic buildings with 4-gonal apartments in [16]. Existence of surface subgroups in fundamental groups of higher-dimensional complexes is discussed, for example, in [7].

In [11] the authors studied surface subgroups of the 23 torsion-free groups acting simply transitively on the vertices of hyperbolic triangular buildings of the smallest non-trivial thickness by searching periodic apartments of genus 2. They showed, that only in the first two groups listed in [10] such an apartment exists, and therefore there also exists a surface subgroup of genus 2. Here we take another approach. Instead of studying the group on the hyperbolic building,

we study first the 1-vertex polyhedron glued from the 15 triangles, and then the 3-cover of it, a 3-vertex polyhedron. We show that these polyhedra admit surface subgroups only in the same two cases out of 23, that were obtained in [11].

2 Notations

Let us consider the triangular complex X glued together from 15 triangles with words on their boundary (for example as in Table 1) . We relax on the condition of orientability, and consider the vector space C_2 freely spanned by the set of triangles of the triangular complex X , the ground field being $\frac{\mathbb{N}}{2\mathbb{N}}$. Thus the vectors in C_2 are sums of certain triangles (with coefficient 1). Now we also have the vector space C_1 freely spanned (on $\frac{\mathbb{N}}{2\mathbb{N}}$) by the set of edges of X . The boundary map $\partial : C_2 \rightarrow C_1$ is defined on the basis of triangles by the relation : $\partial T = a + b + c$ where a, b, c are the three edges in the boundary of the triangle T . Thus, a 2-cycle (mod. 2) is an element of the kernel of ∂ .

The triangular complex X is of thickness 3. This implies that any (non zero) 2-cycle corresponds to the immersion of a surface. Since we chose the ground field to be $\frac{\mathbb{N}}{2\mathbb{N}}$ instead of \mathbb{N} , the immersed surface may be non-orientable.

In order to really get a (quasi-convex) surface subgroup we need also a geometric condition. One sufficient condition is that all local cycles are of length 8. We might allow a bit more, then the immersed surface would not necessarily be isometrically embedded anymore, only a quasi-isometric embedding.

We will divide the search of surface subgroups in the 23 groups into two cases: first we consider cycles in the 1-vertex polyhedron defined by the triangles, then in the 3-vertex cover of this polyhedron.

3 Surfaces immersed to the 1-vertex polygon

We calculated the 2-cycles (mod 2) in the one vertex case for all the 23 torsion free groups (that is, 23 sets of 15 triangles that each form the 1-vertex polygon, listed in the appendix II of [10]) by going through all 2^{15} possible chains. It turned out that some of them are indeed cycles (mod 2). The cycles are listed in Table 2. The notation "Presentation nb 1: 6-cycle 101101000000011" means, that in the first polygonal presentation obtained in [10], there is one 2-cycle, it is of length 6, and it is formed by taking the 1st, 3rd, 4th, 6th, 14th, and 15th triangle in T_1 (see Table 1 for the triangles).

To see whether the 2-cycles come from a surface subgroup, we need the geometric condition. To any cycle mod 2 there is a naturally associated compact triangulated surface together with an immersion inside the triangular complex. Each triangle is equipped with the regular hyperbolic metric with angle $\pi/4$ at vertices. Therefore if the initial cycle was locally an union of 8-cycles, it forces the immersion to be a local isometry, and it follows that the immersion is π_1 -injective, because we are in the context of negative curvature.

Thus we know that if the 2-cycle intersects with the link along 1-cycles of length 8, it corresponds to a surface. This may be too strong condition, but at least it gives some examples.

Table 2: 2-cycles in the 1-vertex polyhedra

Presentation nb 1	6-cycle	101101000000011
Presentation nb 2	6-cycle	000011100010110
	6-cycle	101101000000011
	8-cycle	101110100010101
Presentation nb 3	8-cycle	000011011110101
Presentation nb 4	8-cycle	000011011110101
Presentation nb 5	6-cycle	000011001100011
	8-cycle	101101010001110
	10-cycle	101110011101101
Presentation nb 6	8-cycle	001101110101100
	6-cycle	100000001111001
	10-cycle	101101111010101
Presentation nb 7	6-cycle	000010110011100
	8-cycle	001101110101100
	6-cycle	001111000110000
	6-cycle	100000001111001
	8-cycle	100010111100101
	10-cycle	101101111010101
	8-cycle	101111001001001
Presentation nb 8	6-cycle	101100010000011
Presentation nb 9	8-cycle	000001111111010
Presentation nb 10	8-cycle	100000110110111
Presentation nb 11	8-cycle	100001100111101
Presentation nb 12	8-cycle	100011000011111
Presentation nb 13	6-cycle	100011000111000
Presentation nb 14	8-cycle	100011011100110
Presentation nb 15	10-cycle	101110011010111
Presentation nb 16	6-cycle	100000110000111
Presentation nb 18	6-cycle	000000001111011
Presentation nb 20	6-cycle	001101001100100

The first candidate for a surface subgroup is in the presentation T_9 : the 2-cycle of the triangles (x_8, x_7, x_3) , (x_{10}, x_{13}, x_3) , (x_8, x_5, x_4) , (x_{14}, x_{14}, x_4) , (x_{10}, x_{12}, x_5) , (x_7, x_{12}, x_6) , (x_{15}, x_9, x_6) , (x_{15}, x_{13}, x_9) has a vertex-link consisting of 3 circles of length 8. Thus the surface has 8 regular hyperbolic triangles with angles $\pi/4$, and around the vertex the triangles are situated in such a way that the geometry condition is fulfilled. A surface subgroup consisting of the same triangles was found in T_9 with a completely another method in also [11].

Unfortunately, in all the other 22 cases the 2-cycles have vertex-links other than a set of cycles of length 8, and thus we cannot be sure of the existence of any other surface subgroups.

4 Surfaces immersed to the 3-vertex cover

Let us consider the 3-covers Y_i of the polyhedra X_i with one vertex. We construct the 3-covers as in [3]: We substitute each triple of the form (x_i, x_j, x_k) by three triplets (x_i^1, x_j^2, x_k^3) , (x_i^2, x_j^3, x_k^1) and (x_i^3, x_j^1, x_k^2) , and each (x_i, x_j, x_j) similarly by three triplets (x_i^1, x_j^2, x_j^3) , (x_i^2, x_j^3, x_j^1) and (x_i^3, x_j^1, x_j^2) . We then have 45 triangles, which represent the generators of the index 3 subgroup. Since the 1-vertex polygon X_i consists of 15 triangles, the 3-cover Y_i is glued together from 45 triangle faces. To find surface subgroups we first search for all 2-cycles (mod 2) in the 3-vertex polyhedron Y_i .

In the 3-vertex space Y , we first pick some vertex v . Then all triangles of Y are adjacent to v . So Y is obtained by identifying some boundary edges of a complex B which is a 1-ball around v : in other words B is the star of a vertex v_0 in the universal cover, so B is the cone with base the generalized 4-gon L and apex v . We note that since each edge in Y is contained in three triangles, the identifications on the boundary L of B is very simple: it must identify edges three by three.

Now having a 2-cycle c in Y is the same as having a 2-chain b in B , whose boundary is contained in L (so b is locally a 2-cycle at v), such that moreover the projection of b is c - so the free faces of the 2-chain b cancel pairwise. This is easy to express only in terms of the graph L . The 2-chain b corresponds to a unique 1-chain w in L . The condition that c is a 2-cycle exactly means that

- w is a 1-cycle: it's just a union of circles of the graph L ,
- the identification of the edges on L identifies the edges of w two by two: when e is an edge of L appearing in w , it is identified in L with two other edges e', e'' , and exactly one of e', e'' belongs to w , the other one is exterior to w .

In order to find all possible such 1-cycles we first write L as the quotient of a rooted tree T which is a ball of radius 4 in the trivalent tree. So L is obtained by an identification of points at ∂T . All 1-cycles in the link are generated by 16 generators (cycles of length 8 in the link), easily visible from the rooted tree: each generator goes from one branch to another through the root.

Then we calculate which of the 2^{16} combinations of these 16 generators give a 1-cycle w which corresponds to a 2-cycle b in the ball B , and thus to a 2-cycle c in Y .

With the help of a computerized algorithm, we find several such cycles for each of the 23 presentations.

Since we have a 2-cycle (mod 2) in Y_i , the cover of degree 3 of X_i , it means we have an immersion of a (not necessarily orientable) surface S_i into Y_i . Composing with the covering map $p_i : Y_i \rightarrow X_i$ we get an immersion $i : S_i \rightarrow X_i$. Now we have to insure that we have a local isometry of non-positively curved complexes, i.e. the map is a π_1 -injection, to guarantee that we have found surface subgroup acting on the building.

In this context a local isometry of a triangular surface S_i to the triangular complex X_i is a locally injective simplicial map $f_i : S_i \rightarrow X_i$, such that the image of the circle $\text{link}(S_i, v)$ is a circle of $\text{link}(X_i, f_i(v))$ with no short cut of length $< \pi$, when each edge of the link receives the length $\pi/4$. So the condition means

that if there is a path of less than 4 edges between two vertices of $f_i(\text{link}(S_i, v))$ then this path is inside the circle $f_i(\text{link}(S_i, v))$.

When we go through all the 23 torsion free groups that act simply transitively on the vertices of hyperbolic triangular buildings of the smallest non-trivial thickness, we find 2-cycles in most of them. In six cases the link at least at one of the three vertices is a collection of cycles of length 8. These are presented in the Appendix. In three of the cases the links at all vertices are cycles of length 8. One of them is the same candidate for a surface subgroups in the group G_9 that we found already in the 1-vertex polyhedron. The other two cases, where surface subgroups are four, are the same two groups G_1 and G_2 for which the authors in [11] find periodic planes of genus 2 and therefore also surface subgroups of genus 2.

Group G_1

We find a 2-cycle in the 3-vertex cover. It consists of 8 triangles: (x_1^3, x_{10}^2, x_1^1) , (x_{15}^3, x_2^2, x_1^1) , (x_{15}^2, x_2^1, x_1^3) , (x_{11}^3, x_9^2, x_2^1) , (x_{14}^1, x_3^3, x_2^2) , $(x_{15}^2, x_{13}^1, x_3^3)$, $(x_{14}^1, x_{15}^3, x_9^2)$ and $(x_{13}^1, x_{11}^3, x_{10}^2)$. On the 3-vertex polygon, the link at each vertex is a circle of length 8. On the 1-vertex polygon the image of this 2-cycle under the covering map has therefore a link which is a collection of 3 cycles of length 8. Since the complete vertex-link is a generalized 4-gon and has girth 8, there cannot be any shortcuts between the vertices of the cycles. Thus we have a local isometry, and therefore there exists a surface subgroup in the group G_1 .

This surface subgroup is not the only one, with the method we find two other surface subgroups in G_1 as well. These too come from 8 triangles each, and the links at each vertex are cycles of length 8.

Group G_2

Also in G_2 we find a surface subgroup. It comes from the same triangles (x_1^3, x_{10}^2, x_1^1) , (x_{15}^3, x_2^2, x_1^1) , (x_{15}^2, x_2^1, x_1^3) , (x_{11}^3, x_9^2, x_2^1) , (x_{14}^1, x_3^3, x_2^2) , $(x_{15}^2, x_{13}^1, x_3^3)$, $(x_{14}^1, x_{15}^3, x_9^2)$ and $(x_{13}^1, x_{11}^3, x_{10}^2)$ in the 3-cover as in the previous case. This is sure to be a surface, since the links at the vertices are cycles of length 8. Again, this is not unique. There are two other sets of 8 triangles in T_2 that give a surface subgroup.

Group G_9

For G_9 we already found a surface from the 1-vertex polygon. The corresponding surface is naturally found from the 3-cover, too. On the 3-cover the surface consists of 24 triangles, since every triangle from the 1-vertex polygon is presented three times here.

Group G_{14}

24 triangles in the 3-cover give at one of the vertices a link that consists of 3 cycles of length 8, but at another vertex a cycle of length 24. Such a long cycle always has a shortcut somewhere: take two points from the link that have in this cycle four other points between them. Since the diameter of the generalized 4-gon is four, there is always a path of length four between these points. Thus

the local isometry is violated and this 2-cycle does not correspond to a surface subgroup.

Group G_{18}

16 triangles in the 3-cover form a cycle, but even though at two of the vertices the links consists of 2 cycles of length 8, at the third vertex it is just one cycle of length 16. Thus, similarly as above, this 2-cycle does not correspond to a surface subgroup.

Group G_{20}

24 triangles in the 3-cover form a cycle, but at one of the vertices the link is a cycle of length 24, at another it consists of cycles of lengths 16 and 8, and only in one vertex the link is a combination of cycles of length 8. In the long cycles there are definitely some shortcuts, so this 2-cycle does not correspond to a surface subgroup.

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Appendix: Cycles in 3-vertex polyhedra

Here the labels x_i^j , $i = 1, \dots, 15$, $j = 1, 2, 3$, of the sides of the triangles are given as integers from 1 to 45, $x_i^j \triangleq 3(i-1) + j$.

Number 1

Triangles in the 2-cycle:

3 29 1
45 5 1
44 4 3
33 26 4
40 9 5
44 37 9
40 45 26
37 33 29

The cycles are generated by:

9 40 45 1 3 4 33 37

Cycles in the link:

1 3 4 33 37 9 40 45

Triangles in the 2-cycle:

2 28 3
43 6 2
44 4 3
32 25 6
42 8 4
43 39 8
42 44 25
39 32 28

The cycles are generated by:

6 7 45 1 3 28 39 43

6 7 45 1 3 4 42 25

Cycles in the link:

3 4 42 25 6 43 39 28

Triangles in the 2-cycle:

1 30 2
45 5 1
43 6 2
31 27 5
41 7 6
45 38 7
41 43 27
38 31 30

The cycles are generated by:

27 31 30 1 3 28 39 43

6 7 45 1 3 28 39 43

Cycles in the link:

1 30 31 27 43 6 7 45

Number 2

Triangles in the 2-cycle:

3 29 1
45 5 1
44 4 3
33 26 4
40 9 5
44 37 9
40 45 26
37 33 29

The cycles are generated by:

9 40 45 1 3 4 33 37

Cycles in the link:

1 3 4 33 37 9 40 45

Triangles in the 2-cycle:

2 28 3
43 6 2
44 4 3
32 25 6
42 8 4
43 39 8
42 44 25
39 32 28

The cycles are generated by:

6 7 45 1 3 28 39 43

6 7 45 1 3 4 42 25

Cycles in the link:

3 4 42 25 6 43 39 28

Triangles in the 2-cycle:

1 30 2
45 5 1
43 6 2
31 27 5
41 7 6
45 38 7
41 43 27
38 31 30

The cycles are generated by:

27 31 30 1 3 28 39 43

6 7 45 1 3 28 39 43

Cycles in the link:

1 30 31 27 43 6 7 45

Number 9

Triangles in the 2-cycle:

24 20 7
22 21 8
23 19 9
30 38 7
28 39 8
29 37 9
24 14 10
22 15 11
23 13 12
42 41 10
40 42 11
41 40 12
30 35 13
28 36 14
29 34 15
21 35 16
19 36 17
20 34 18
45 26 16
43 27 17
44 25 18
45 38 25
43 39 26
44 37 27

The cycles are generated by:

15 7 30 1 3 28 39 34
15 7 30 1 3 4 33 22
24 7 30 1 3 4 42 10
27 13 30 1 3 28 39 43
27 13 30 1 3 4 33 37
12 13 30 1 3 4 42 40
21 16 45 1 3 4 33 22
9 25 45 1 3 28 36 19
9 25 45 1 3 4 33 37
18 25 45 1 3 28 39 34

Cycles in the link:

7 24 10 42 40 12 13 30
9 19 36 28 39 43 27 37
15 22 21 16 45 25 18 34

Number 14

Triangles in the 2-cycle:

2 43 3
28 6 2
29 4 3
33 8 4

32 7 6
 20 10 9
 45 35 7
 43 36 8
 44 34 9
 24 17 10
 22 18 11
 34 27 11
 22 39 14
 40 24 14
 21 26 16
 19 27 17
 36 32 16
 35 31 18
 30 38 19
 28 39 20
 29 37 21
 40 30 26
 45 38 31
 44 37 33

The cycles are generated by:

6 7 45 1 3 43 39 28
 21 7 45 1 3 43 36 16
 21 7 45 1 3 4 33 37
 9 31 45 1 3 43 36 10
 9 31 45 1 3 4 33 34
 18 31 45 1 3 43 39 22
 27 19 30 1 3 4 33 34
 24 40 30 1 3 43 36 10

Cycles in the link:

3 4 33 37 21 16 36 43
 6 7 45 31 18 22 39 28
 9 10 24 40 30 19 27 34

Number 18

Triangles in the 2-cycle:

33 11 4
 32 10 6
 42 8 4
 41 7 6
 27 14 7
 25 15 8
 42 23 10
 40 24 11
 17 34 15
 43 39 14

19 27 17
 33 29 19
 40 36 23
 41 34 24
 39 32 25
 43 36 29

The cycles are generated by:

6 28 45 1 3 43 39 7
 6 28 45 1 3 4 42 10
 27 31 30 1 3 43 39 7
 27 31 30 1 3 4 33 19
 15 34 30 1 3 43 39 25
 24 34 30 1 3 43 36 40

Cycles in the link:

4 33 19 27 7 6 10 42
 15 25 39 43 36 40 24 34

Triangles in the 2-cycle:

31 12 5
 32 10 6
 40 9 5
 41 7 6
 27 14 7
 26 13 9
 42 23 10
 41 22 12
 16 36 14
 45 38 13
 21 26 16
 32 28 21
 42 35 22
 40 36 23
 38 31 27
 45 35 28

The cycles are generated by:

9 13 45 1 3 43 36 40
 6 28 45 1 3 43 39 7
 6 28 45 1 3 4 42 10
 21 28 45 1 3 43 36 16
 12 31 30 1 3 4 42 22
 27 31 30 1 3 43 39 7

Cycles in the link:

6 7 27 31 12 22 42 10
 9 13 45 28 21 16 36 40

Number 20

Triangles in the 2-cycle:

3 29 1

2 28 3
 45 17 1
 43 18 2
 9 20 4
 7 21 5
 36 23 4
 34 24 5
 13 12 8
 14 10 9
 33 41 7
 31 42 8
 45 38 10
 44 37 12
 27 44 13
 26 43 15
 28 36 14
 29 34 15
 40 33 17
 41 31 18
 24 38 19
 23 37 21
 42 26 19
 40 27 20

The cycles are generated by:

12 13 30 1 3 28 36 37
 27 13 30 1 3 16 33 40
 24 34 30 1 3 16 42 19
 15 34 30 1 3 28 39 43
 9 10 45 1 3 28 36 4
 18 10 45 1 3 28 39 43
 18 10 45 1 3 16 42 31
 21 25 45 1 3 28 36 37
 21 25 45 1 3 16 33 7

Cycles in the link:

1 3 28 36 4 9 10 45
 7 33 40 27 13 12 37 21
 15 34 24 19 42 31 18 43

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